Assignment of Chapter 4

1. The generator matrix of a $(7, 3)$ linear block code is

$$\boldsymbol{G} = \begin{pmatrix} 0 & 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 1 & 0 \end{pmatrix}.$$

   (1) Please determine the parity-check matrix of this code.

   (2) Please construct a systematic generator matrix of this code.

   (3) Please determine the minimum distance and the error-correcting capability of the code.

   (4) Discuss the relationship between this code and the $(7, 4)$ Hamming code.

2. Let $g(x) = 1 + x^2 + x^3 + x^4$ be the generator polynomial of a $(7, 3)$ cyclic code.

   (1) Please determine its parity-check matrix.

   (2) Please determine its systematic generator matrix.

   (3) Please determine the codebook generated by $g(x)$ and the minimum Hamming distance.

3. Let n and k be integers such that $n \geq k$. Let $\mathbf{G}$ and $\mathbf{H}$, of dimension $k \times n$ and $(n-k) \times n$, respectively, be the generator and parity-check matrices for a linear code $\mathcal{C} \in \mathbb{F}_2^n$. Is the $n \times n$ matrix $\begin{pmatrix} \mathbf{G} \\ \mathbf{H} \end{pmatrix}$ necessarily invertible? Please prove it or show a counterexample.